B.Sc. (Honours) Examination, 2018
Semester-III
Statistics
Course: CC-7
Mathematical Analysis Theory (Theory and Tutorial)

Time : 3 Hours **Full Marks : 60**

Questions are of value as indicated in the margin

Group-A (Answer any ten questions) 10x1=10

1. What do you mean by an ordered field?
2. How do you represent $\sqrt{3}$ on directed line?
3. Write down the definition of a dense set and give an example.
4. Discuss the statement: 'every bounded sequence is convergent'.
5. State Raabe's test for convergence of a series.
6. The series $\sum_{n=1}^{\infty} \cos\left(\frac{n+1}{n^2}\right)$ is convergent?
7. What do you mean by an unconditionally convergent series.
8. Find $\lim_{x\to 0} Sgn(x)$.
9. Define indicator function. How do you write a step function using it.
10. Discuss the extremum of $x - [x]$ at $x = 0$.
11. If $f(x)$ is a polynomial of degree n. Then what is the value of $\Delta^{n+1} f(x)$.
12. Which kind of interpolation formula we use for interpolating near the middle of the tabular values with equal space?
13. What do you mean by inverse interpolation?
14. Graphically show that $[a, b]$ and $[c, d]$ are equivalent sets.

Group-B (Answer any five questions) 5x6=30

15. (a) Show that $7^n - 3^n$ is divisible by 4.
    (b) If $a \in \boldsymbol{R}$ is positive, then $\exists\, n \in \mathbf{N}$ such that $n-1 \leq a < n$. 3+3
16. (a) Show that $\{(n+1)^{1/n}\}$ converges to 1.
    (b) Show that the limit of a convergent sequence is unique.
    (c) In usual notation show that $\Delta - \nabla = \delta^2$. 2+2+2

P.T.O.

17. (a) If $\{x_n\}$ and $\{y_n\}$ are convergent sequences that converges to l and n respectively, then show that $\{|x_n - y_n|\}$ converges to $|l-m|$.
(b) Show that $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is a convergent sequence. 3+3

18. (a) If $\sum u_n$ be a convergent series of positive real numbers, then show that $\sum \frac{u_n}{1+u_n}$ is also convergent.
(b) Test the convergence of the series $\frac{1}{1+2^{-1}} + \frac{1}{1+2^{-2}} + \frac{1}{1+2^{-3}} + \cdots$.
(c) Derive the error in Trapezoidal rule. 2+2+2

19. (a) Prove that the series $\frac{a}{b} + \frac{a(a+c)}{b(b+c)} + \frac{a(a+c)(a+2c)}{b(b+c)(b+2c)} + \cdots$, $a, b, c > 0$ is convergent if $b > a+c$ and divergent if $b \leq a+c$.
(b) Discuss the convergence of $\sum \frac{1}{n \log n (\log \log n)}$. 3+3

20. (a) Let $f : (0, \infty) \to \boldsymbol{R}$ be defined by $f(x) = x(e^{\frac{1}{x^3}} - 1 + \frac{1}{x^3})$. Then find out correct statement(s). (i) $\lim\limits_{x\to\infty} f(x)$ exists. (ii) $\lim\limits_{x\to\infty} xf(x)$ exists. (iii) $\lim\limits_{x\to\infty} x^2 f(x)$ exists. (iv) There exists $m > 0$ such that $\lim\limits_{x\to\infty} x^m f(x)$ does not exists.
(b) Show that the Dirichlet function is discontinuous everywhere.
(c) Check whether $f(x) = x^2$ is uniformly continuous on $\boldsymbol{R}$ or, not. 2+2+2

21. (a) If $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \ldots + \frac{a_n}{n+1} = 0$, where $a_i \in \boldsymbol{R}$, show that the equation $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = 0$ has at least one real root between $(0, 1)$.
(b) Show that $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$ has only one real root, provided $a, b, c \in \boldsymbol{R}$.
(c) Let $f : (0, \infty) \to \boldsymbol{R}$ be given by $f(x) = log(x) - x + 2$. Then the number of roots of f is (i) 0 (ii) 1 (iii) 2 (iv) 3. 2+2+2

22. (a) Find the solution of the difference equations $x(n+1) = ax(n) + b,\ x(0) = x_0$.
(b) In usual notation show that
$$u_1 + u_2 + u_3 + \cdots + u_n = \binom{n}{1} u_1 + \binom{n}{2} \Delta u_1 + \cdots + \Delta^{n-1} u_1$$
3+3

Group-C (Answer any two questions) 2x10=20

23. (a) State and prove Maclaurin's theorem for expansion of a function. Hence obtain the power series expansion of e^x.
(b) Find the global extrema of the function $f(x) = xe^{-x}$ for $x \in [0.1, 5]$. (4+3)+3

24. (a) State and prove Lagrange's interpolation formula.

(b) Hence show that the sum of Lagrangian functions is unity and it is invariant under linear transformation.

(c) Show that for equispaced arguments $[x_0, x_1, ..., x_n] = \dfrac{\Delta^n y_0}{n!h^n}$. 3+(2+2)+3

25. (a) If $x_1, x_2 > 0$ and sequence $\{x_n\}$ is defined as $x_{n+1} = \frac{1}{2}(x_n + x_{n-1})$, $n > 1$. Then show that $x_1, x_3, x_5, \cdots$ are monotonic decreasing and $x_2, x_4, x_6, \cdots$ are monotonic increasing and they converge to $\frac{1}{3}(x_1 + 2x_2)$.

(b) State and prove Cauchy condensation test. Hence discuss the convergence of $\sum \frac{1}{n^p}$, $p > 0$. 5+(3+2)